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FORTRAN PROGRAM FOR CALCULATING THE TRAJECTORY OF PROJECTILES

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16. Abstract This report deals with a FORTRAN program for numerical calculation of projectiles' trajectories with two degrees of aeromechanical freedom. Dupuis' law of air resistance is applied under ÖB [expansion unknown] standard atmospheric conditions. The program selects a time step with regard to initial velocity and air resistance. The program then adapts this time step to the trajectory continually during the entire operation. The program costs less than 1 kr/trajectory, and is therefore suitable when many trajectories are to be calculated. Agreement with firing tables is extremely good, and the program error is entirely within the accuracy of the tables.

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SYMBOLS

α	Time step constant	(1)
C	Speed of sound	(m/s)
C_D	Dupuis constant for projectile	(m/s^2)
d	Measurement of calculational accuracy	(m)
dx/dt	Horizontal velocity coordinate	(m/s)
dy/dt	Vertical velocity coordinate	(m/s)
ε	Factor	(1)
F	Drag function dependent upon Mach number	
ϕ	Trajectory tangent's angle to horizontal plane	(°)
ϕ_0	Elevation	(°)
g	Gravitational acceleration	(m/s^2)
h	Time setp	
j	Form factor for projectile	
K_r	Force perpendicular to Earth's surface	(N)
K_θ	Force parallel with Earth's surface	(N)
m	Cycle index (m = 1, 2, 3)	(1)
M	Mach number = V/C	(1)
n	Time step index (n = 1, 2, 3)	(1)
p	Pressure	(N/m^2)
p_0	Reference pressure	(N/m^2)
r	Polar position coordinate	(m)
S	x coordinate for impact in plane y = 0	(m)
θ	Angle in polar coordinate system	(rad)
t	Time	
U	$p/p_0\ C_D\ F(M,j)/V$	-
v	Velocity	(m/s)
v_0	Initial velocity	(m/s)
x	Horizontal position coordinate	(m)
y	Vertical position coordinate	(m)
y_0	Initial position above ground plane	(m)

FORTRAN PROGRAM FOR CALCULATING THE TRAJECTORY OF PROJECTILES

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Assignment Number D4 23 F /1*

Recipients: MHZ (2 rx), FortF, FMV-A (5 ex), FMV-M (2 ex), FMV-f (2 ex), FFV/Hk (2 ex), AB Borfors (4 ex), LIAB/Lindesberg, FOA 1 (2 ex), FOA 4 (2 dx. FOA 2: 00, 07, 08, 20, 21, 24, 50, 54 (3 ex), 56 (3 ex), 57 (3 ex), 70 (3 ex), FRÖ.

Introduction /4

Certain FOA projects have demanded numerous and expensive computations of various projectile's trajectories where only the initial velocity, the Dupuis coefficient, and the projectile's form factor have varied, assuming that the projectile is stable.

This report deals with a FORTRAN program in which the projectile is assumed to be a point moving in ÖB [expansion unknown] standard atmosphere under the influence of Dupuis' law of air resistance. Two degrees of aeromechanical freedom are allowed. Consideration has been made for the curvature of the Earth, and the accuracy of the program is within that of the artillery firing tables.

Since one must consider the effects of air force in calculations of trajectory motion, the motion equations are not analytically solvable unless numerical solutions can be applied. This report is a description of the theoretical background, difference equations and the computer program.

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* Numbers in the margin indicate pagination in the foreign text.

Equations for Trajectory Motion

The motion equations presented here allow for the curvature of the earth. The Coriolis effect, whose influence upon trajectory motion depends upon the firing direction, is ignored. Polar coordinates (r, θ), with the datum at the earth's center, have been used. The coordinate system (x,y) is defined as:

$$\begin{cases} x = R\theta \\ y = r-R \end{cases} \tag{1}$$

Thus the altitude above the earth's surface is y, while x is a coordinate on the surface. The motion equations in the coordinate system (r, θ) are:

$$\begin{cases} \dfrac{d^2r}{dt^2} - r\left(\dfrac{d\theta}{dt}\right)^2 = \dfrac{K_r}{m} \\ r^2\dfrac{d^2\theta}{dt^2} + 2\,r\,\dfrac{dr}{dt}\,\dfrac{d\theta}{dt} = r\,\dfrac{K_\theta}{m} \end{cases} \tag{2}$$

We now include the air force upon the projectile $-m\,f(v)$, and assume that this force is applied in the trajectory tangent's direction. With the inclusion of the gravitational acceleration, we then derive:

$$\begin{cases} K_r = -\,m\,f(v)\,\dfrac{dr}{dt}/v\ -mg \\ K_\theta = -\,m\,f(v)\,r\,\dfrac{d\theta}{dt}/v \end{cases} \tag{3}$$

where

$$v = \left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2\right]^{1/2} = \left[\left(\frac{dy}{dt}\right)^2 + \left(1+\frac{y}{R}\right)^2\left(\frac{dx}{dt}\right)^2\right]^{1/2}. \tag{4}$$

In the (x,y) system the motion equations are:

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= -f(v)\frac{dx}{dt}/v - 2\frac{dx}{dt}\frac{dy}{dt}/(R+y) \\ \frac{d^2y}{dt^2} &= -f(v)\frac{dy}{dt}/v - g + \left(\frac{1}{R} + \frac{y}{R^2}\right)\left(\frac{dx}{dt}\right)^2 \end{aligned}\right\} \quad (5)$$

Air Forces

At time t = 0, the projectile is assumed to depart from the point x = 0, $y = y_o$. Its velocity is v_o, and its direction of travel corresponds to the angle of elevation ϕ_o. Retardation of the projectile due to drag is described according to Dupuis $f(v) = C_D \cdot p/p_o \cdot F(M,j)$ where C_D is a constant, p is the pressure, p_o is a reference pressure, M is the Mach number and j is a form factor. When $1 < M$, we have, according to reference 1:

$$F(M,j) = 96000(M-j-0,5) + 15936(M-2,05) \cdot 10^{-5,5(M-1,94)^2} \quad (6)$$

when $0,83931 < M \leq 1$ the following is true:

$$F(M,j) = 96000(0,14-0,35\ j+(0,36-0,65\ j) \cdot 10^{-5,8(1-M)}) \quad (7)$$

when $0,81736 < M \leq 0,83931$ the following is true:

$$F(M,j) = 96000(0,14+0,36 \cdot 10^{-5,8(1-M)}) - 58054\ j\ M^2 \quad (8)$$

and when $M \leq 0,81736$ the following is true:

$$F(M,j) = (24630 - 58054\ j)\ M^2 \quad (9)$$

Environmental data have been taken from reference 2.

The speed of sound C and the relative pressure p/p_o when $y \leq 10,000$ m are:

$$C = 20,052 \sqrt{278 - 0,006\,y}\ \mathrm{m/s} \tag{10}$$

$$p/p_o = \left(1 - \frac{0,006\,y}{278}\right)^{5,7} \tag{11}$$

and when $y > 10,000$ m

$$C = 20,052 \sqrt{218}\ \mathrm{m/s} \tag{12}$$

$$p/p_o = 0,250119 \cdot e^{-1,5688 \cdot 10^{-4}(y-10000)} \tag{13}$$

For gravitational acceleration:

$$g(y) = 9,818 \left(1 - \frac{2\,y}{6,37 \cdot 10^{6}}\right)\ \mathrm{m/s^2} \tag{14}$$

Numerical Method of Solution /7

The velocity is calcualted with Heun's method, which yields an error $0(h^3)$ in one time step. An initial value (index f) for dx/dt (t+h) and dy/dt (t+h) is derived from:

$$\frac{dx}{dt_f}(t+h) = \frac{dx}{dt}(t) - h\,\frac{dx}{dt}(t)\left[U(t) + \frac{2}{R+y(t)}\,\frac{dy}{dt}(t)\right]$$

$$\frac{dy}{dt_f}(t+h) = \frac{dy}{dt}(t) - h\left[U(t)\,\frac{dy}{dt}(t) + g(y(t)) - \left(\frac{dx}{dt}(t)\right)^2\left(\frac{1}{R} + \frac{y(t)}{R^2}\right)\right] \tag{15}$$

Thereby:

$$U(t) = p/p_o \cdot F(M,j) \cdot C_D/v(t) \tag{16}$$

An initial value of $y_f(t+h)$ for y at time t+h is derived from:

$$y_f(t+h) = y(t) + \frac{h}{2}\left[\frac{dy}{dt}(t) + \frac{dy}{dt_f}(t+h)\right]. \tag{17}$$

Using these data, the function $U_f(t+h)$ is calculated analagously to (16), after which the final velocity values are derived from:

$$\frac{dx}{dt}(t+h) = \frac{dx}{dt}(t) - \frac{h}{2}\left[U(t)\,\frac{dx}{dt}(t) + U_f(t+h)\,\frac{dx}{dt_f}(t+h) + \right.$$
$$\left. + \frac{2}{R+y(t)}\left(\frac{dx}{dt}(t)\frac{dy}{dt}(t) + \frac{dx}{dt_f}(t+h)\frac{dy}{dt_f}(t+h)\right)\right] \tag{18}$$

$$\frac{dy}{dt}(t+h) = \frac{dy}{dt}(t) - \frac{h}{2}\left[U(t)\frac{dy}{dt}(t) + U_f(t+a)\,\frac{dy}{dt_f}(t+h) - \right.$$
$$\left. - \left(\frac{1}{R} + \frac{y(t)}{R^2}\right)\left(\left(\frac{dx}{dt}(t)\right)^2 + \left(\frac{dx}{dt_f}(t+h)\right)^2\right)\right] -$$
$$- h\, g(y(t)).$$

The final position is determined by: /8

$$\left.\begin{aligned} x(t+h) &= x(t) + \frac{h}{2}\left[\frac{dx}{dt}(t) + \frac{dx}{dt}(t+h)\right] \\ y(t+h) &= y(t) + \frac{h}{2}\left[\frac{dy}{dt}(t) + \frac{dy}{dt}(t+h)\right] \end{aligned}\right\} \tag{19}$$

This formula also has an error $O(h^3)$ in one time step.

The time step should be increased succesively during the operation, since the velocity and the retardation forces tend to vary considerably. The following formula has been used:

$$h=\alpha \min \begin{cases} \dfrac{2 \cdot \frac{dy}{dt}(t=0)}{g(y=0)} \\ \\ \dfrac{v}{C_D \cdot p/p_o \cdot F(M,j)} \end{cases} \qquad (20)$$

The first expression in equation 20 corresponds to a situation in which the air force can be ignored.

The last expression in equation 20 corresponds to the characteristic velocity decrease interval in a situation where the air force is the dominating retardation mechanism. The factor α is selected small enough that satisfactory computational accuracy is achieved.

As v exceeds the speed of sound, a special formula for the time step must be used, since F(M,j) as per equations 1-4 varies discontinuously. The following procedures have been used when v(t+h) > C and v(t+h) < C, and when v(t) < C and v(t+h) > C. h has been derived from equation 20, and a new h value (h') is calculated as follows:

$$h' = h \cdot \frac{M(t) - 1}{M(t) - M(t+h)} \qquad (21)$$

The calculation is repeated from t till t+h' (at t+h' the velocity v is extremely close to C). After the sound barrier has been broken, equation 20 is once again used in selection of the time step.

The calculations are terminated when $y(t_n+h) < 0$. The impact /9
time $t_n+\varepsilon h$ in the plane $y = 0$ is determined by using:

$$\varepsilon = \frac{y(t_n)}{y(t_n) - y(t_n+h)} \qquad (22)$$

and the horizontal position of the impact is determined by:

$$S = x(t_n+\varepsilon h) = x(t_n) + \varepsilon h \frac{dx}{dt} (t=t_n) \qquad (23)$$

After a trajectory has been calculated, the α value in equation 20 is halved, and the calculation is repeated. This process continues until the difference between the two results $|S_{m+1} - S_m| < d$, where d is an input constant and $m = 1,2,3...$. The error in S_{m+1} is then on the order of d/3.

Computer Program

The main routine processes input, output and most of the computational work according to equations 14-23. Subroutine FUNK is employed in calculation of the drag function F(M,j) using equations 6, 7, 8 or 9, depending upon the Mach number which is applicable. The Mach number, the relative pressure and the velocity are calculated using equations 10, 11 and 4 or 12, 13 and 4, using the MACH subroutine. The trajectories for a considerable number of projectiles can be calculated in the same run, and the respective projectiles' data decks are given out in order. FORTRAN lists for the routines are given in annexes 2-4.

Input and Output

SI units are used for all input and output. The input data is punched according to the form in annex 1, which clearly indicates those data which are to be entered.

The time step constant is 0.005-0.05.

The table data is written on unit 6, and each individual trajectory (point for point) is written on unit 8.

Examples of output data are given in annex 5, which includes tables and diagrams of several trajectories.

When the program is executed with the object deck, a CPU time of approximately 0.6 seconds per 100 steps is needed in IBM 360/75's high-speed storage.

Final Comments /10

As shown, the points of impact generated by this program exhibit small differences after the time step has been halved only a few times (see annex 5). There is excellent agreement with the firing tables given in annex 5, which show the results of test firings of 105 mm high explosive shells (m/60Z). According t firing table (F1092-051920 SKJT 1 10.5 cm HAUB, page 281), t firing range for v = 610 m/s is 121 1/2 hm, ϕ = 28.57° (500'), j = 0.04 and C_D = 3.213 x 10^{-4} with lng 8 [expansion unknown]. Our calculated radius of action is 121.43 hm. The calculations were carried out in relatively few time steps because of the variable time step. This resulted in low costs, approximately 0.80 kr/trajectory for approximately 50 time steps, which allow for satisfactory accuracy in most cases.

The program is stored in source program form, and in object form on punched cards numbered G 312 and G 313 (for production runs), in Grindsjön's data terminal, where data forms as per annex 1 are available. Approximately 36 k bytes of memory are needed in runs with the object deck.

REFERENCES

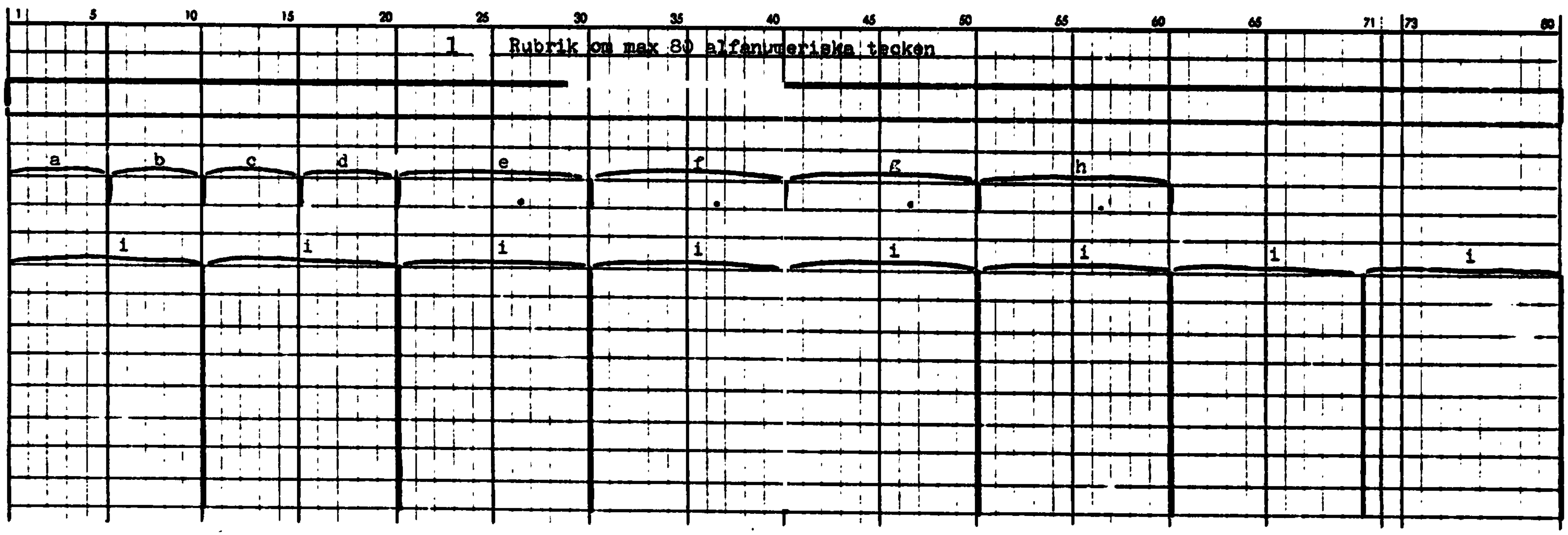
1. MHS Compendium in Exterior Ballistics, Royal Swedish Armed Forces Staff College, 1962.

2. Ballistic Symbols FSD A0100:2, The Standardization Delegation of the Swedish Armed Forces.

Annex 1:1(1). Input for program for calculating projectile trajectories. /11

Composite table on unit 06 and each individual trajectory on unit 08.

All units in SI system



Key. 1. Caption for maximum of 80 alphanumerical symbols

(I 5)	a. Number of times time step is halved	(F 10.3)	f. Initial altitude above ground
(I 5)	b. Number of different velocities for which calculation can occur (maximum 20)	(F 10.3)	g. Maximum allowable difference in calculated impact after 1 or more time step divisions
(I 5)	c. Number of different elevations (max. 20)	(F 10.3)	h. Projectile constant (j in firing tables)
(I 5)	d. Number of different CD_o constants (max. 20)	(E 10.3)	i. Initial velocity(s), elevation(s) and CD_o constants written with B or F^o format. (o = Dupuis' constant)

PROGRAM FOR CALCULATION OF PROJECTILE TRAJECTORIES WITH TWO DEGREES AEROMECHANICAL FREEDOM. DUPUIS' LAW OF AIR RESISTANCE IS APPLIED.

THE PROGRAM SELECTS A TIME STEP AND ADAPTS IT TO THE TRAJECTORY CONTINUALLY.

INPUT

CARD 1	TXT	20A4	CAPTION FOR MAX 80 ALPHANUMERICAL SYMBOLS
CARD 2	MMAX	I5	NUMBER OF TIMES TIME STEP IS HALVED
	IHA	I5	NUMBER OF DIFFERENT VELOCITIES FOR WHICH CALCULATION WILL OCCUR
	IVI	I5	NUMBER OF VARIOUS ELEVATIONS FOR WHICH CALCULATION WILL OCCUR
	ICD	I5	NUMBER OF VARIOUS CDO CONSTANTS FOR WHICH CALCULATION WILL OCCUR
	ALFA1	F10.0	TIME STEP CONSTANT (SUITABLE VALUE = 0.005-0.05)
	YO	F10.0	INITIAL ALTITUDE ABOVE GROUND FOR PROJECTILE
	D	F10.0	MAXIMUM ALLOWABLE DIFFERENCE IN IMPACTS AFTER ONE OR MORE TIME STEP DIVISIONS
	S	F10.0	FORM FACTOR (CORRESPONDING TO J IN FIRING TABLES)
CARD 3	V01	E10.3	INITIAL VELOCITY(S) (IHA ST)
	V1	E10.3	ELEVATION(S) (IVI ST) WRITTEN DIRECTLY AFTER V01
	CDA	E10.3	CDO CONSTANT(S) (ICD ST) WRITTEN DIRECTLY AFTER V1

READOUT

EACH INDIVIDUAL TRAJECTORY READ OUT STEP FOR STEP ON UNIT 8, WITH A COMPOSIT TABLE ON UNIT 6

SUBROUTINES INVOLVED: MACH, FUNK

ALL UNITS IN SI SYSTEM

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```
      DIMENSION TXT(20),V01(20),VI(20),CDA(20)
      DATA RAD,ST,GY/57.29578,3.14E-7,6.37E6/
      CALL DATE(IYEAR,IMONTH,IDAY,IDATE)
    1 READ(5,2,END=31)(TXT(K),K=1,20),MMAX,IHA,IVI,ICD,ALFA1,YO,D,S
    2 FORMAT(20A4/4I5,4F10.0)
      DO 3 K=1,20
      V01(K)=0.
```

```
 .VI(K)=0.
3 CDA(K)=0.
  N=MAX0(IHA,IVI,ICD)
  READ(5,4)(VO1(K),K=1,IHA),(VI(K),K=1,IVI),(CDA(K),K=1,ICD)
4 FORMAT(8E10.3)
  WRITE(6,5)MMAX,IHA,IVI,ICD,ALFA1,YO,D,S,(VO1(K),VI(K),CDA(K),     /13
 *K=1,N)
5 FORMAT(1H1,5X,'INPUT DATA      (SI - UNITS  .'/6X,27('-')//6X,
 *'NUMBER OF TIME STEP HALVINGS           =',I6//6X,
 *'NUMBER OF VELOCITIES                   =',I6//6X,
 *'NUMBER OF ELEVATIONS                   =',I6//6X,
 *'NUMBER OF CDO CONSTANTS                =',I6//6X,
 *'TIME STEP CONSTANT                     =',F10.3//6X,
 *'INITIAL ELEVATION ABOVE GROUND         =',F10.3//6X,
 *'ALLOWABLE DIFFERENCE IN IMPACTS        =',F10.3//6X,
 *'FORM FACTOR FOR PROJECTILE             =',F10.3//8X,
 *'VELOCITIES,     ELEVATIONS,      CDO CONSTANTS    '/6X,45('-')/(6X,
 *F13.3,F15.3,E1(.5/))
  WRITE(6,6)IYEAR,IMONTH,IDAY,(TXT(K),K=1,20)
6 FORMAT(1H1,10X,'TRAJECTORY CALCULATIONS OF PROJECTILES COMPOSITE
 *  TABLE',3I3/11X,65('+')//11X,20A4/11X,80('+')
 *               //16X,'START',15X,' NUMBER   OPERATION-    MAX-     OPERATION
 *-  CD-COEF -     IMPACT -     IMPACT - FORM FACTOR'
 *               /14X,'VELOCITY      ELEVATION      STEP     DISTANCE   CLIMB ALTITUDE
 *     TIME   FICIENT    ANGLE        VELOCITY          J'/11X,103('+'))
  DO 30 IC=1,ICD
  CD=CDA(IC)
  DO 30 IH=1,IHA
  VO=VO1(IH)
  DO 30 IV=1,IVI
  FI=VI(IV)
  RANGE=0.
  N=0
  ALFA=ALFA1
7 WRITE(8,8)(TXT(L),L=1,20)
8 FORMAT(1H1,10X,'TRAJECTORY CALCULATIONS OF PROJECTILES', 20A$/11X,107('+')//
 *4X,' TIME      PATH       ALTITUDE    ANGLE       VELOCITY I   VELOCITY 1
 *  FINAL VELOCITY 1    MACH'/3X,'(SEK)   ',2(7X,'(M)'),4X,'I TRAJECTORY X-L
 *COORDINATE M/S)  Y-COORDINATE (M/S)  TRAJECTORY M/S)  NUMBER'/1X,88('+'))
  FIX=FI


 FIA=FIX/RAD
 V=VO
 N=0
 TT=0.
 X=0.                                                               /14
 YMAX=0.
 DXDT=VO*COS(FIA)
 DYDT=VO*SIN(FIA)
 Y=YO
```

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```
    9 H2=DYDT/4.909
      CALL MACH(Y,DXDT,DYDT,U,O,V)
      WRITE(8,10)TT,X,Y,FIX,DXDT,DYDT,V,U
   10 FORMAT(1X,F10.5,3F10.2,4F12.2)
      GO TO 12
   11 WRITE(8,10)TT,X,Y,FIX,DXDT,DYDT,V,U
   12 CALL FUNK(U,S,F)
      R=CD*O*F/V
      U1=U
      K=1
      H=ALFA*AMIN1(1./R,H2)
      DX=DXDT
      DY=DYDT
      GR=9.818*(1.-Y*ST)
      RYT=(1.+Y/RY)/RY
   13 DXDT=DX*(1.-H*(R+2.*DY/(RY+Y)))
      DYDT=DY-H*(R*DY+GR-DX*DX*RYT)
      YN=Y+H*(DY+DYDT)/2.
      CALL MACH(YN,DXDT,DYDT,U,O,V)
      CALL FUNK(U,S,F)
      RN=CD*O*F/V
      DXDT=DX-H*((R*DX+RN*DXDT)/2.+(DX*DY+DXDT*DYDT)/(RY+Y))
      DYDT=DY-H*((R*DY+RN*DYDT-(DX*DX+DXDT*DXDT)*RYT)/2.+GR)
   16 YH=H*(DY+DYDT)/2.
      YN=Y+YH
      CALL MACH(YN,DXDT,DYDT,U,O,V)
      GO TO(17,21),K
   17 IF(U1-1.)19,18,18
   18 IF(U-1.)20,21,21
   19 IF(U-1.)21,20,20
   20 H=H*(U1-1.)/(U1-U)+1.E-04
```

/15

```
      K=2
      GO TO 13
   21 TA=TT
      FIX=RAD*ATAN2(DYDT,DXDT)
      TT=TT+H
      XR=X
      X=X+H*(DX+DXDT)/2.
      YP=Y
      Y=Y+YH
      YMAX=AMAX1(YMAX,Y)
      N=N+1
      IF(Y)22,22,11
   22 EPS=YP/(YP-Y)
      RANGE1=RANGE
      Y=0.
      X=XR+H*EPS*DX
      TT=TA+H*EPS
      DXDT=DX+EPS*(DXDT-DX)
      DYDT=DY+EPS*(DYDT-DY)
      CALL MACH(Y,DXDT,DYDT,U,O,V)
      FIX=RAD*ATAN2(DYDT,DXDT)
```

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```
      RANGE=X
      WRITE(8,10)TT,X,Y,FIX,DXDT,DYDT,V,U
      IF(M-1)23,25,25
   23 WRITE(6,24)V0,FI,N,RANGE,YMAX,TT,CD,FIX,V,S
   24 FORMAT(11X,F10.1,F10.1,I10,2F10.1,F10.1,E12.5,2F10.1,F10.3)
      GO TO 27
   25 WRITE(6,26)N,RANGE,YMAX,TT,FIX,V
   26 FORMAT(31X,I10,2F10.1,F10.1,12X,2F10.1)
   27 IF(M)29,29,28
   28 IF (ABS(RANGE1-RANGE)-D)30,29,29
   29 ALFA=0.5*ALFA
      M=M+1
      IF(M-MMAX)7,30,30
   30 CONTINUE
      GO TO 1
   31 STOP
      END
```

/16

```
C.....................................................................
      SUBROUTINE FUNK(U,S,F)
C
C     CALCULATE DRAG FUNCTION FOR PROJECTILE
C
C     VARIABLES IN LIST OF ARGUMENTS
C
C     U      MACH NUMBER
C     S      FORM FACTOR (CORRESPONDS TO J IN FIRING TABLES)
C     F      DRAG FUNCTION CALCULATED BY ROUTINE
C
C.....................................................................
C
      IF(U-1.)2,2,1
C
C     FOR MACH NO.  .GT. 1.
C
    1 A=-5.5*(U-1.94)**2
      F=9.6E4*(U-S-0.5)+1.5936E4*(U-2.05)*10.**A
      RETURN
    2 U2=U*U
      S5=5.8054E4*S
      IF(U-0.83931)4,4,3
C
C     FOR MACH NO.  .GT. 0.83931 OCH .LE. 1.
C
    3 A=-5.8*(1.-U)
      F=9.6E4*(0.14-0.35*S+(0.36-0.65*S)*10.**A)
      RETURN
    4 IF(U-0.81736)6,6,5
```

```
C
C      FOR MACH NO.  .GT. 0.81736 OCH .LE. 0.83931
C
     5 A=-5.8*(1.-U)
       F=9.6E4*(0.14+0.36*10.**A)-S5*U2
       RETURN
C
C      FOR MACH NO.  .LE. 0.81736
C
     6 F=U2*(2.463E4-S5)
       RETURN
       END
```

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```
C.............................................................
      SUBROUTINE MACH(Y,DXDT,DYDT,U,Q,V)
C
C     CALCULATES RELATIVE PRESSURE, MACH NUMBER, AND VELOCITY
C
C     VARIABLES IN LIST OF ARGUMENTS
C
C     Y      ALTITUDE ABOVE SEA LEVEL FOR PROJECTILE
C     DXDT   VELOCITY'S HORIZONTAL TIME DERIVATIVE
C     DYDT   VELOCITY'S VERTICAL TIME DERIVATIVE
C     U      MACH NUMBER
C     Q      RELATIVE PRESSURE
C     V      PROJECTILE VELOCITY
C
C.............................................................
C
      A=Y-1.E4
      IF(A)2,2,1
C
C     ALTI-
C     TUDE .GT. 10000 M
C
    1 C=296.061
      Q=0.250119*EXP(-A*1.5688E-4)
      GO TO 3
C
C     ALTI-
C     TUDE .LE. 10000 M
C
    2 T=1.-Y*2.158273E-5
      C=334.33*SQRT(T)
      Q=T**5.7
    3 V=SQRT(DYDT*DYDT + DXDT*DXDT*(1. + Y/6.37E6)**2)
      U=V/C
      RETURN
      END
```

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ANNEX 5:1(2). TRAJECTORY CALCULATIONS FOR PROJECTILES,
COMPOSITE TABLE 72 8 9

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105 MM HIGH EXPLOSIVE SHELL M/60Z LNG 8

INITIAL VELOCITY	ELEVATION	NUMBER OF TIME STEPS	RANGE	MAXIMUM CLIMBING ALTITUDE	FLIGHT TIME	CD COEFFI-CIENT	IMPACT ANGLE	IMPACT VELOCITY	FORM FACTOR J
610.0	28.6	23	12147.9	2268.6	41.8	0.32130E-03	-43.8	272.1	0.040
		44	12146.2	2272.4	41.8		-43.8	272.2	
		87	12143.4	2274.1	41.8		-43.8	272.3	
		172	12143.1	2274.0	41.8		-43.8	272.3	
		343	12142.7	2274.0	41.8		-43.8	272.3	

ANNEX 5:2(2). TRAJECTORY CALCULATIONS FOR PROJECTILES
105 MM HIGH EXPLOSIVE SHELLS M/60 Z LNG 8

TIME (SEC)	RANGE	ALTITUDE (M)	ANGLE IN TRAJECTORY	VELOCITY IN X COORDINATE (M/S)	VELOCITY IN Y COORDINATE (M/S)	FINAL VELOCITY IN TRAJECTORY (M/S)	MACH NUMBER
0.0	0.0	1.50	28.57	535.72	291.72	610.00	1.82
0.78933	412.79	223.31	27.92	510.19	270.30	577.39	1.73
1.61821	825.57	438.74	27.18	485.81	249.50	546.16	1.64
2.48408	1236.17	646.04	26.37	462.60	229.32	516.36	1.56
3.38838	1644.52	844.57	25.46	440.54	209.76	487.98	1.47
4.33603	2052.06	1034.34	24.45	419.58	190.74	460.96	1.39
5.33361	2460.69	1215.35	23.30	399.66	172.16	435.23	1.32
6.38840	2872.26	1387.33	22.01	380.73	153.93	410.74	1.25
7.50823	3288.53	1549.64	20.55	362.73	135.95	387.45	1.18
8.70203	3711.35	1701.30	18.87	345.62	118.13	365.34	1.11
9.98042	4142.80	1840.95	16.94	329.37	100.35	344.41	1.05
11.21643	4541.42	1955.06	14.95	315.64	84.30	326.80	1.00
12.70655	5002.32	2067.46	12.39	302.98	66.56	310.30	0.95
14.90877	5655.46	2187.61	8.34	290.18	42.56	293.39	0.90
17.85443	6491.06	2268.55	2.56	277.16	12.39	277.54	0.85
20.82571	7297.91	2262.24	-3.58	265.94	-16.64	266.55	0.82
23.79706	8072.70	2171.34	-9.89	255.58	-44.54	259.51	0.80
26.76828	8817.39	1999.16	-16.20	245.68	-71.36	255.91	0.78
29.73956	9533.07	1748.90	-22.36	236.04	-97.09	255.29	0.78
32.71085	10220.20	1423.92	-28.24	226.48	-121.66	257.13	0.78
35.68213	10878.84	1027.82	-33.76	216.85	-144.96	260.87	0.79
38.65341	11508.63	564.60	-38.86	207.07	-166.84	265.93	0.80
41.62469	12109.01	38.72	-43.52	197.05	-187.14	271.75	0.81
41.82210	12147.91	0.0	-43.81	196.37	-188.37	272.11	0.81

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